

GIANT RESONANCES IN COULOMB EXCITATIONS OF RELATIVISTIC IONS

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We propose a soluble model to incorporate the nonlinear effects in the transition probabilities of the multiphonon Giant Dipole Resonances based on the SU(1,1) algebra. Analytical expressions for the multi-phonon transition probabilities are derived. For reasonably small magnitude of nonlinearity $x \simeq 0.1 - 0.2$ enhancement factor for the Double Giant Resonance excitation probabilities and the cross sections reaches values $1.3 - 2$ compatible with experimental data.

1 Introduction

Coulomb Excitation in collisions of relativistic ions is one of the most promising methods in modern nuclear physics^{1,2,3,4,5}. One of the most interesting applications of this method to studies of nuclear structure is the possibility to observe and study the multi phonon Giant Resonances¹. In particular, the double Dipole Giant Resonances (DGDR) have been observed in a number of nuclei^{6,7,8}. The “bulk properties” of the one- and two-phonon GDR are now partly understood¹ and they are in a reasonable agreement with the theoretical picture based on the concept of GDR-phonons as almost harmonic quantized vibrations.

Despite that, there is a persisting discrepancy between the theory and the data, observed in various experiments^{6,7,8,9,10} that still remains to be understood: the double GDR excitation cross sections are found enhanced by factor $1.3 - 2$ with respect to the predictions of the harmonic phonon picture^{1,3,11,12}. This discrepancy, which almost disappears at high bombarding energy, has attracted much attention in current literature^{4,13,14,15,16,17,22,23,24}; among the approaches to resolve the problem are the higher order perturbation theory treatment¹⁷, and studies of anharmonic/nonlinear aspects of GDR dynamics^{4,22,23,24}. Recently, the concept of hot phonons^{15,16} within Brink-Axel mechanism was proposed that provides microscopic explanation of the effect. These seemingly orthogonal explanations deserve clarification which we try to supply here.

The purpose of this work is to examine, within a soluble model the role of the nonlinear effects on the transition amplitudes that connect the multiphonon states in a heavy-ion Coulomb excitation process. Most studies of anharmonic

corrections^{22,23,24} concentrated on their effect in the spectrum^{25,26}.

Within our model, the nonlinear effects are described by a single parameter, and the model contains the harmonic model as its limiting case when the nonlinearity goes to zero. We obtain analytical expressions for the probabilities of excitation of multiphonon states which substitute the Poisson formula of the harmonic phonon theory. For the reasonable values of the nonlinearity, the present model is able to describe the observed enhancement of the double GDR cross sections quoted above.

Having in mind to show how analytical results follows from the nonlinear model, and to explain how the model works, we restrict ourselves here to its simplest version (transverse approximation, or SU(1,1) dynamics) and keep numerics up to minimum level. We postpone till further publications detailed numerical analysis and comparison with the data. Microscopic origins of the nonlinear effects (which are considered here phenomenologically) are also beyond the scope of our discussion.

2 Excitation of multi-phonon GDR: Coupled-channels problem

We work in a semiclassical approach¹¹ to the coupled-channels problem, i.e., the projectile-target relative motion is approximated by a classical trajectory and the excitation of the Giant Resonances is treated quantum mechanically¹³. The use of this method is justified due to the small wavelenghts associated with the relative motion in relativistic heavy ion collisions. The separation coordinate is treated as a classical time dependent variable, and the projectile motion is assumed to be a straight line².

The intrinsic dynamics of excited nucleus is governed by a time dependent quantum Hamiltonian (see Refs. ^{11,12}). The intrinsic state $|\psi(t)\rangle$ of excited nucleus is the solution of the time dependent Schrödinger equation

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = [H_0 + V(t)]|\psi(t)\rangle, \quad |\psi(t)\rangle = \sum_{N=0} a_N(t)|N\rangle \exp(-iE_N t), \quad (1)$$

where H_0 is the intrinsic Hamiltonian and V is the channel-coupling interaction. We let $\hbar = 1$, $c = 1$. The standard coupled-channel problem is to find the expansion amplitudes $a_N(t)$ in the wave packet $|\psi\rangle$ as functions of impact parameter b where E_N is the energy of the state $|N\rangle$ specified by the numbers of excited GDR phonons N . We assume the colliding nuclei to be in their ground states before the collision. The amplitudes obey the initial condition $a_N(t \rightarrow -\infty) = \delta_{N,0}$ and they tend to constant values as $t \rightarrow \pm\infty$ (the interaction $V(t)$ dies out at $t \rightarrow \pm\infty$). The excitation probability of an intrinsic state $|N\rangle$ in a collision with impact parameter b and the total cross

section for excitation of the state $|N\rangle$ (given by the integral over the impact parameter) are

$$W_N(b) = |a_N(\infty)|^2, \quad \sigma_N = 2\pi \int_{b_{gr}}^{\infty} b W_N(b) db \quad (2)$$

(we use the grazing value $b_{gr} = 1.2(A_{exc}^{1/3} + A_{sp}^{1/3})$ as the lower limit). Hereafter, the labels *exc* (*sp*) refer to the excited (spectator) partner in a colliding projectile-target pair. We neglect the here nuclear contribution²¹ to the excitation process.

It is convenient to treat the coupled channel equations (1) in terms of the unitary operator U_I (the interaction picture):

$$i \frac{d}{dt} U_I(t) = V_I(t) U_I(t), \quad V_I(t) = e^{iH_0 t} V(t) e^{-iH_0 t}, \quad U_I(t = -\infty) = I, \quad (3)$$

where the time-dependent Hamiltonian $H(t) = H_0 + V(t)$ that acts in the intrinsic multi-GDR states is given by $H_0 = \omega \hat{N}_d$, $\hat{N}_d \equiv \sum_m d_m^+ d_m^-$ and

$$V(t) = v_1(t)[(E1_{-1})^\dagger - (E1_{+1})^\dagger] + v_0(t)(E1_0)^\dagger + \text{Herm. Conj.} \quad (4)$$

where $E1_m^\dagger$ and $E1_m$ are the dimensionless operators acting in the internal space of the multi-GDR states. The functions v are given in², e.g.,

$$v_1(t) = \frac{w}{[1 + (\frac{\gamma v}{b} t)^2]^{3/2}}, \quad w = \rho \frac{Z_{sp} e^2 \gamma}{2b^2} \sqrt{\frac{N_{exc} Z_{exc}}{A_{exc}^{2/3} m_N \cdot 80 \text{MeV}}}. \quad (5)$$

Here, m_N and e are the proton mass and charge, Z , N and A denote the nuclear charge, the neutron number and the mass number of the colliding partners, $\gamma = (1 - v^2)^{-1/2}$ is relativistic factor, v is the velocity and the parameter ρ is the deal of the strength absorbed by the collective motion (usually assumed to be close to unity)¹.

In the harmonic approximation, the operators $E1_m^\dagger, E1_m$ are given by the GDR phonon creation and destruction operators of corresponding angular momentum projection m , $E1_m^\dagger = d_m^\dagger$. This model of “ideal bosons” coupled linearly to the Coulomb field has well known exact nonperturbative solution (see, e.g.¹²) for the excitation probabilities

$$W_N = e^{-|\alpha^{harm}|^2} \frac{|\alpha^{harm}|^{2N}}{N!},$$

$$|\alpha^{harm}|^2 = \sum_{m=0,\pm 1} |\alpha_m^{harm}|^2 = 2|\alpha_1^{harm}|^2 + |\alpha_0^{harm}|^2, \quad (6)$$

i.e., the Poisson formula where the amplitudes α_m^{harm} are expressed in terms of the modified Bessel functions. At the colliding energies sufficiently high, the longitudinal contribution ($\propto |\alpha_0^{harm}|^2$) is suppressed by a factor proportional to γ^{-2} , see, e.g.,³. In the following, we will work in this “transverse approximation” dropping the longitudinal term (the results are still qualitatively valid at lower energies).

3 Nonlinear Model

Our idea is to keep the spectrum of GDR system harmonic with the Hamiltonian $H_0 = \omega N$. That is supported by the systematics of the observed DGDR energies, E_2 , which yields $E_2 \simeq (1.75 - 2)\omega^1$. The conclusion on the weak anharmonicity in the spectrum follows also from theoretical considerations^{25,26}.

The transition operators $E1^\dagger, E1$ that couple intrinsic motion to the Coulomb field can however include nonlinear effects: one can use the boson-expansion-like expression for them

$$E1_m^\dagger = d_m^\dagger + x d_{m_1}^\dagger d_{m_2}^\dagger d_{m_3} + x_1 d_{m'_1}^\dagger d_{m'_2}^\dagger d_{m'_3}^\dagger + \dots \quad (7)$$

where the parameters x_i determines the strengths of the nonlinear contributions. In particular, this could be a result of nonlinearities (anharmonicities) in the phonon Hamiltonian obtained in higher orders of perturbation theory. A reasonable way to take into account these nonlinear effects that we adopt in this work is to keep in the expansion (7) infinite series of type

$$\begin{aligned} E1_m^\dagger &= d_m^\dagger + x \sum_{m_1} d_m^\dagger d_{m_1}^\dagger d_{m_1} - \frac{x^2}{2} \sum_{m_1 m_2} d_m^\dagger d_{m_1}^\dagger d_{m_1} d_{m_2}^\dagger d_{m_2} + \dots = \\ &= d_m^\dagger \left(1 + 2x\hat{N}_d\right)^{1/2}, \end{aligned} \quad (8)$$

where the single parameter $x > 0$ determines the strength of the nonlinear effects, i.e., the problem reduces to the harmonic oscillator with linear coupling when $x \rightarrow 0$. Nonzero values of x that we will consider here lead to a number of nonlinear effects that results in enhancement of the excitation cross sections for the double Giant resonances.

Expression (8), together with the standard coupled-channel formalism presented in the previous section, composes our nonlinear model. Let us mention its advantageous points:

- (i) The model takes into account the nonlinear effects (not all of them by at least part of them), nonlinearity is governed by a single parameter
- (ii) The model is exactly soluble beyond the perturbative theory

(iii) Its results reproduce correctly the magnitude of the *enhancement factor* for the Double GDR cross sections and its dependence on bombarding energy

4 Algebraic solution: SU(1,1) dynamics

To solve the highly nonlinear problem (3) with (4) and (8) we introduce the triad of operators D^+ , D^- and D^0 that are given by

$$D^- = \sqrt{k + \frac{1}{2}\hat{N}_d}(d_{+1} - d_{-1}), \quad D^+ = \frac{1}{2^{1/2}}(d_{+1}^+ - d_{-1}^+) \sqrt{2k + \hat{N}_d}, \\ D^0 = \frac{1}{4} \left[(d_{+1}^+ - d_{-1}^+)(d_{+1} - d_{-1}) + 2(2k + \hat{N}_d) \right], \quad (9)$$

where D^+ the Hermitean conjugate $D^+ = (D^-)^\dagger$ with $N_d \equiv d_{+1}^+ d_{+1} + d_{-1}^+ d_{-1}$ and $k \equiv (4x)^{-1}$. It is easy to check that they obey the commutation relations for the *noncompact* SU(1,1) algebra

$$[D^-, D^0] = D^-, \quad [D^+, D^0] = -D^+, \quad [D^-, D^+] = 2D^0. \quad (10)$$

i.e., Eqs.(9) can be viewed a version of the Holstein-Primakoff realization of SU(1,1) algebra. The parameter $k = 1/(4x)$ is related to the Casimir operator of SU(1,1).

The dynamics of the system, in the transverse approximation, can be expressed now in terms of the operators D^+ , D^- and D^0 (9) only. The interaction-picture evolution equation (3) and its formal exact solution, the time-ordered exponential (see, e.g.,³¹) than read

$$i \frac{d}{dt} U_I(t) = \left[\frac{v_1(t)}{\sqrt{k}} e^{i\omega t} D^+ + \frac{v_1(t)}{\sqrt{k}} e^{-i\omega t} D^- \right] U_I(t), \\ U_I(t) = T \exp \left(-i \int_{-\infty}^t dt' V_I(t') \right) \quad (11)$$

where the commutation relation $[\hat{N}_d, D^\pm] = \pm D^\pm$ and the algebra (10) has been used in (3),(4) and (8).

From purely mathematical viewpoint, the problem described by the last equation drops into the universality class of the systems with SU(1,1) dynamics that can be analyzed by means of generalized coherent states^{29,30} for the SU(1,1) algebra. (For other algebraic approaches to scattering problems, see Refs.^{19, 20}).

Due to closure of the commutation relations between the operators D^+ , D^- and D^0 that enter the exponential in Eq.(11), the time-ordered exponential can be represented in another equivalent form that involve ordinary operator exponentials only (see, e.g.,¹⁸):

$$U_I(t) = \exp\left[\frac{\alpha(t)}{\sqrt{k}}D^+\right]\exp\left[\left[\log\left(1 - \frac{|\alpha(t)|^2}{k}\right) - i\phi(t)\right]D^0\right]\exp\left[-\frac{\alpha^*(t)}{\sqrt{k}}D^-\right] \quad (12)$$

and some time-dependent complex number $\alpha(t)$ (star means complex conjugation) and real number $\phi(t)$ (phase)²⁹. The unknown functions $\alpha(t)$ and $\phi(t)$ can be found from simple differential equations which relate them to the function $v_1(t)$ in the Hamiltonian $H(t)$. These equations can be restored after substituting the right hand side of Eq.(12) into the left hand side of the Schrödinger equation for the operator $U_I(t)$ (11) and collecting the terms which have the same operator structure. Proceeding this way, we obtain, after some algebraic manipulations, from (11) with using the commutation relations (10) and applying the known formula formula for the operator exponentials

$$e^{\hat{Y}}\hat{X}e^{-\hat{Y}} = \hat{X} + [\hat{Y}, \hat{X}] + \frac{1}{2!}[\hat{Y}, [\hat{Y}, \hat{X}]] + \dots$$

the following Riccati-type equation for the complex amplitude α :

$$i\frac{d}{dt}\alpha = v_1(t)e^{i\omega t} + 4xv_1(t)e^{-i\omega t}\alpha^2. \quad (13)$$

The phase $\phi(t)$ is given by a simple integral

$$\phi(t) = (2/k) \int_{-\infty}^t dt_1 \text{Re}(v_1(t_1)\alpha(t_1)e^{-i\omega t_1}).$$

In fact, $\phi(t)$ does not contribute to W_N and σ_N . The simple nonlinear equation (13) accounts for all orders of quantum perturbation theory for the problem Eqs.(3),(4),(11). It is also seen from Eq.(12) that unitarity is automatically preserved within present formalism ($U_I^\dagger = U_I^{-1}$).

The expression for the amplitudes $a_N(t)$ which we are interesting in follows from (12) immediately after projection of the state

$$|\psi(t)\rangle = U_I(t)|0\rangle$$

onto the intrinsic states with definite number of GDR phonons, N . From Eq.(12), we have

$$U_I(t)|0\rangle = e^{-ik\phi(t)} \left(1 - \frac{|\alpha(t)|^2}{k}\right)^k \exp\left[\frac{\alpha(t)}{\sqrt{k}}D^+\right]|0\rangle.$$

From the last equation, we obtain the final expression for the amplitude $|a_N(\infty)|$ and the excitation probabilities for the excited states with N phonons:

$$W_N = |a_N(\infty)|^2,$$

$$|a_N(\infty)| = (1 - 4x|\bar{\alpha}(x)|^2)^{\frac{1}{4x}} \left(\frac{\Gamma(\frac{1}{2x} + N)}{N! \Gamma(\frac{1}{2x})} \right)^{1/2} (4x|\bar{\alpha}(x)|^2)^{N/2} \quad (14)$$

Here, the quantity $\bar{\alpha}(x)$ is the asymptotic solution to the Riccati equation (13) at $t \rightarrow \infty$ subject to the initial condition $\alpha(-\infty) = 0$. The combination $x^{1/2}|\bar{\alpha}(x)|$ in (14) can be viewed as a “special function” of the two parameters, $x^{1/2}F/\omega$ and the adiabaticity parameter $\frac{\omega b}{v\gamma}$. It can be easily tabulated by solving (13). The cross sections are then obtained from the usual formula (2) with using (14).

The harmonic limit of these results corresponds to the case $x \rightarrow 0$, when the nonlinearity disappears in the transition operators (8) and the coupling to electromagnetic field becomes linear. Then at $x \rightarrow 0$ the last nonlinear term drops from the equation (13), and the amplitude is reduced to its harmonic value

$$|\bar{\alpha}(x)| \rightarrow |\alpha_{\pm 1}^{harm}| = \left| -i \int_{-\infty}^{\infty} v_1(t) e^{i\omega t} dt \right| = 2 \frac{w}{\omega} \left(\frac{\omega b}{v\gamma} \right)^2 K_1 \left(\frac{\omega b}{v\gamma} \right)$$

that is given by the modified Bessel function ¹²¹. The expression for the probabilities W (14) reduces at $k \rightarrow \infty$ to the Poisson formula (6), thus the harmonic results ¹²¹ are restored.

At nonzero nonlinearity $x > 0$, the excitation probabilities W_N (14) for multiple GDR ($N > 1$) turn out to be enhanced as compared to their values in the harmonic limit W_N^{harm} , as illustrated in Fig.1.

The deviation in the excitation probabilities for the N -phonon states W_N (14) from their harmonic values W_N^{harm} (6) (the *enhancement factor*) is given by the ratio

$$\frac{W_N}{W_N^{harm}} = \frac{\Gamma(\frac{1}{2x} + N)}{\Gamma(\frac{1}{2x})} \frac{(1 - 4x|\bar{\alpha}(x)|^2)^{\frac{1}{2x}}}{e^{-2|\alpha_1^{harm}|^2}} \frac{|\bar{\alpha}(x)|^{2N}}{|\alpha_1^{harm}|^{2N}} \geq 1. \quad (15)$$

The following ratio is especially representative (see (6),(14)) for the two-phonon case

$$R_2 = \frac{W_2}{W_2^{harm}} / \frac{W_1}{W_1^{harm}} = (1 + 2x) \frac{|\bar{\alpha}(x)|^2}{|\alpha_1^{harm}|^2}. \quad (16)$$

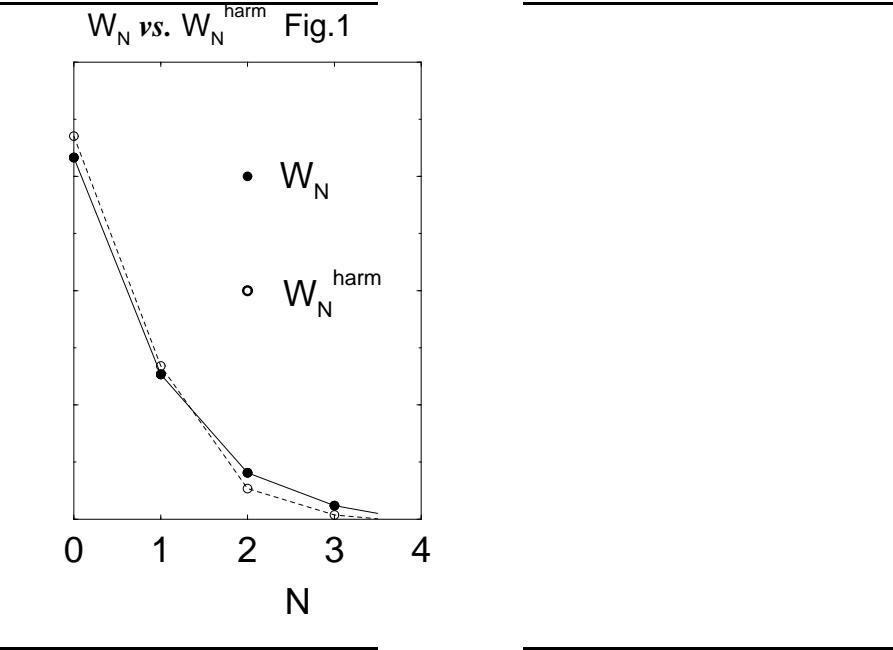


Figure 1: Effect of the nonlinearity on the N -phonon excitation probabilities(schematic plot). Nonlinear model values W_N (dark circles connected by the solid line) as compared to the harmonic model (Poisson) results W_N^{harm} (empty circles connected by the dashed line).

The first factor in this expression, $1 + 2x$, results from the kinematic enhancement of the transition probabilities due to nonlinear effects considered here. The last factor in (15),(16) results from dynamical effects caused by linearity which are incorporated in the asymptotic solution of the nonlinear equation (13). Unlike the first factor that depends on x only, the second one depends on the bombarding energy and it gives rise to additional enhancement in low energy domain. The same is valid for the enhancement factor in the cross sections, r_2 , that is given by the formula

$$r_2 = \frac{\sigma_2}{\sigma_2^{\text{harm}}} = \frac{2\pi \int_{b_{gr}}^{\infty} b W_2 db}{2\pi \int_{b_{gr}}^{\infty} b W_2^{\text{harm}} db}. \quad (17)$$

5 Results and discussion

The interesting feature of these results is that the enhancement factor (15),(16) is much more sensitive to the bombarding energy than to the parameters of the spectator partner. This is just what has been observed in experiments: the values of r_2^{exp} found for DGDR in ^{208}Pb projectile using different targets ^{120}Sn , ^{165}Ho , ^{208}Pb , ^{238}U ⁹ are very close to each other and they correlate, within the error bars, with the value

$$r_2(^{208}Pb) \simeq 1.33 , \quad \text{high energy}, \quad \gamma \simeq 1.7 \quad (18)$$

(bombarding energy $\varepsilon \simeq 640$ Mev/per nucleon).

The same picture was found in experiments on Coulomb desintegration of ^{197}Au target using various projectiles ^{20}Ne , ^{86}Kr , ^{197}Au , ^{209}Bi ⁸. Also, the similar conclusion of nearly constant value of r_2 in ^{208}Pb target with various projectiles has been made in work¹⁰ but for low bombarding energy $\varepsilon \simeq 60-100$ Mev/per nucleon. In this case,

$$r_2(^{208}Pb) \simeq 2 , \quad \text{low energy}, \quad \gamma \simeq 1.06 - 1.10. \quad (19)$$

According to the results of calculation within present nonlinear model, these enhancement factors would correspond to the nonlinearity parameter x equal to

$$x(^{208}Pb) \simeq 0.16 - 0.20$$

i.e., to a reasonably small nonlinearity.

Below, we present the exact results for the cross sections calculated according to Eqs.(2), and (14),(17) and with solving Eq.(13) numerically.

The dependence of the enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ (17) for the DGDR excitation on the strength of the nonlinearity x is shown on Fig. 1. for the process $^{208}Pb + ^{208}Pb$ (we use the case of bombarding energy $\varepsilon = 0.64\text{GeV}/\text{per nucleon}$). It is seen that the enhancement factor drops to unity at big values of k (harmonic limit) and grows at stronger nonlinearity. The scaling value of r_2 is also shown for comparison.

In view of the difference between the enhancement factor values at high and low energies (Eqs.(18,19)), it is interesting to trace energy dependence of the enhancement factor.

Deviations in r_2 from the straight line $1 + 2x$ occur at both low and high energies. At $\gamma \rightarrow 1$, adiabatic approximation is valid, and this yields to $|\alpha| > |\alpha_1^{harm}|$. Thus, $R_2 > 1 + 2x$ and $r_2 > 1 + 2x$. By contrast, at higher energies, the dynamical nonlinear effects tend to reduce the magnitude of $|\alpha|$, thus $|\alpha|/|\alpha_1^{harm}| < 1$, and $R_2 < 1 + 2x$. To sum up, the enhancement factor

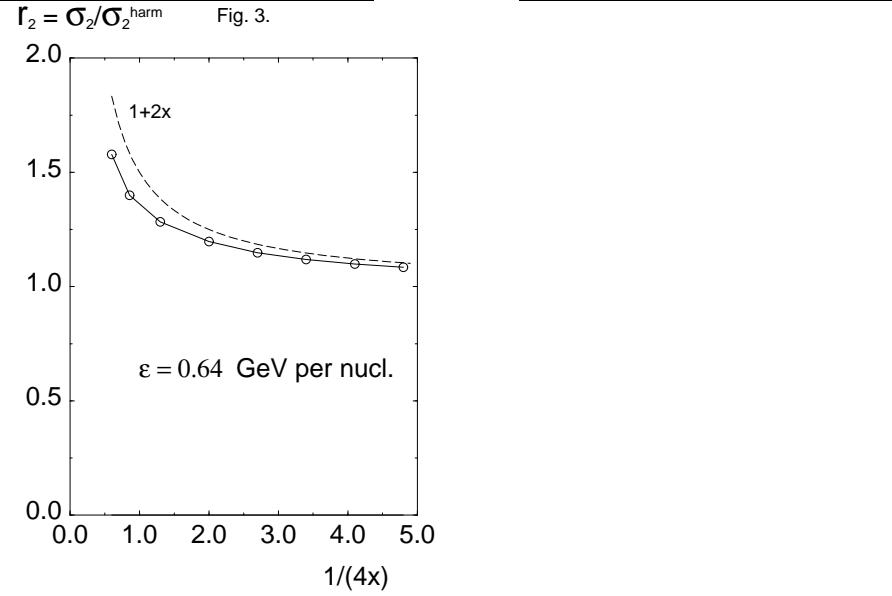


Figure 2: Enhancement factor $r_2 = \sigma_2 / \sigma_2^{\text{harm}}$ for the Double GDR excitation in $^{208}\text{Pb} + ^{208}\text{Pb}$ process at bombarding energy $\epsilon = 640\text{MeV}/\text{per nucleon}$ as a function of the inverse nonlinearity parameter $k = \frac{1}{4x}$ (symbols, solid curve is to guide the eye). Dashed curve corresponds to the scaling $r_2 \approx (1 + 2x) = \frac{2k+1}{2k}$.

for the DGDR excitation cross section, $r_2 = \sigma_2 / \sigma_2^{\text{harm}}$ drops from 2 – 2.5 (for low bombarding energies $\epsilon \sim 100\text{MeV}$ per nucleon) to 1.2 – 1.3 (for $\epsilon \sim 640 - 700\text{MeV}$ per nucleon) while fixed value of nonlinearity x is used.

On Fig.2., we plotted the value of the enhancement factor calculated numerically for the case of $^{208}\text{Pb} + ^{208}\text{Pb}$ process. The magnitude of nonlinearity is kept fixed $x = 0.19$.

It is seen that reasonably small nonlinearity reproduces correctly the observable value of the enhancement factor and its energy dependence. These results are in correspondence with the experimentally observed trends and with microscopic models based on Axel-Brink concept¹⁵.

To conclude, we presented here a simple model that accounts for the nonlinear effects in the transition probabilities for the excitation of multi-phonon Giant Dipole Resonances in Coulomb excitation via relativistic heavy ion collisions. The model is based on the group theoretical properties of the boson

$$r_2 = \sigma_2 / \sigma_2^{\text{harm}}$$

Fig. 2.

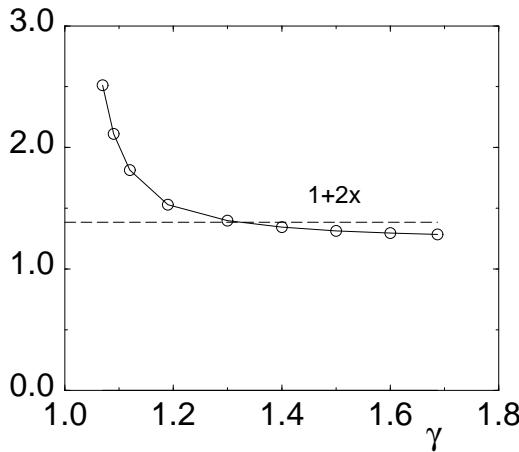


Figure 3: Enhancement factor $r_2 = \sigma_2 / \sigma_2^{\text{harm}}$ for the Double GDR excitation in $^{208}\text{Pb} + ^{208}\text{Pb}$ process as a function of relativistic factor γ (symbols, solid curve is to guide the eye). The value of the nonlinear parameter x is kept to be equal to $x = 0.19$. The scaling value (constant $1 + 2x = \frac{2k+1}{2k}$) is shown by dashed curve.

operators. It allows to construct the solution for the dynamics of the multi-phonon excitation within coupled-channel approach in terms of the generalized coherent states of the corresponding algebras. The exactly solvable harmonic phonon model appears to be a limiting case of the present model when the nonlinearity parameter x goes to zero. The main advantages of the limiting harmonic case (unrestricted multiphonon basis, preservation of unitarity and analytical results in nonperturbative domain) remain present in our nonlinear scheme. Therefore, the model can be viewed as a natural extension of the harmonic phonon model to include the nonlinear effects in a consistent way while keeping the model solvable.

The probabilities to excite double-phonon GDR appear to be enhanced by a factor $(1+2x)(|\bar{\alpha}(x)|^2 / |\alpha_1^{\text{harm}}|^2)$; that results in the cross section enhancement factor. This can be viewed as a hint that the discrepancy between the measured cross-sections of double GDR and the harmonic phonon calculations can be resolved within present nonlinear model by means of using an appropriate value of the nonlinear parameter x for a given nucleus. The experimental

values of enhancement of σ_2 with respect to the harmonic results for the excited ^{208}Pb nucleus are almost insensitive to the details of the collision process. The enhancement factor drops as the bombarding energy grows. This is consistent with the data and gives results similar to those recently obtained in a possibly different context, with a theory based on the concept of fluctuations (damping) and the Brink-Axel mechanism^{15,16,27,28}. It would be certainly worthwhile to establish possible connections between the two approaches.

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